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PROBLEM OF ESTIMATING THE CREEP STRENGTH UNDER STEP LOADING

A. M. Lokoshchenko and S. A. Shesterikov

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In many investigations of the work of structures under variable stress under prolonged high-temperature action conditions, the main question is the possibility of estimating the rupture time from test results under constant stresses. The rule of linear summation of the partial times, proposed in [1] to analyze test results under variable temperature, is ordinarily utilized as the simplest and best known hypothesis. We consider the case when the stress σ_1 in the specimen and the effective temperature t_1 changes by a jump to σ_2 and remains constant for a time t_2 until rupture at the time $t^* = t_1 + t_2$. We write the sum of the partial times in the form

$$A = t_1/t_1^* + t_2/t_2^*. \tag{1}$$

In case the principle of linear summation is satisfied

$$A \equiv 1. \tag{2}$$

Here t_1^* (or t_2^*) is understood to be the time to fracture for stresses σ_1 (or σ_2) invariant during the testing. Many investigations confirm the rule (2) to some extent, however, systematic deviations are observed in a significant quantity of papers, which are outside the boundaries of the natural spread. For certain materials a deviation of A from 1 to one side is hence characteristic, independently of the test parameters, while for other materials the quantity A is greater or less than 1 depending on the sign of the difference $(\sigma_1 - \sigma_2)$.

The behavior of steel EI388 at 600°C was investigated in [2] for $\sigma_1 > \sigma_2$ and $\sigma_1 < \sigma_2$ for small changes in the stress ($|\sigma - \sigma_2|/\sigma_1 < 0.06$), and the tests exhibited a significant one-sided deviation from the law (2): the mean value of A was $A_0 = 0.72$. A model permitting the description of the deviation of A from 1 to one side, independently of the sign of the difference $(\sigma_1 - \sigma_2)$, is proposed below.

The concept of a mechanical equation of state, proposed in [3], is used with a system of kinetic equations within the framework of the mechanics of continuous media to describe the creep of metals, to determine the parameters characterizing the state under consideration. One structural parameter $\omega(t)$ which is a certain measure of the "spalling" of the material, is utilized most frequently to describe the creep strength. A value of ω from the range $0 \leqslant \omega \leqslant 1$, is ascribed to each "spalling" state, where $\omega = 0$ corresponds provisionally to the undamaged material, and $\omega = 1$ corresponds to the time of rupture t^*.

It is known that the nature of rupture for a number of materials at the identical temperature can be qualitatively distinct depending on the stress level. At high stresses the

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development of irreversible shear creep strains is observed until rupture, which passes mainly over the body of the grain. At low stresses pore formation occurs along the grain boundaries, union of the pores grows into cracks which set the material into intragranular rupture. It is evident that under conditions when disturbance of the structure of two kinds exists, it is natural to introduce two structure parameters $\omega(t)$ and $\Omega(t)$.

Let us consider the following kinetic equations characterizing the change in the structure parameters ω and Ω in the time t:

$$\dot{\omega} = B(\sigma/\sigma_0)^n / f'(\omega),\ \Omega = B(\sigma/\sigma_0)^m / F'(\Omega),\ k = (m - n) > 0, \tag{3}$$

where the parameters ω and Ω vary between 0 at the initial instant and 1, $f(\omega)$ and $F(\Omega)$ are differentiable functions of their arguments that grow continuously from the values $f(0) = F(0) = 0$ to $f(1) = F(1) = 1$. The time of rupture is determined by a certain relationship between the parameters ω and Ω. As the simplest rupture condition we take

$$\max(\omega, \Omega) = 1. \tag{4}$$

Relationships (3) and (4) show that the structure parameters $\omega(t)$ and $\Omega(t)$ vary independently in time. Rupture sets in at the time t^* when one of the parameters becomes equal to 1.

We first consider the creep strength under constant stress ($\sigma(t) = \text{const}$). Integrating (3) with respect to the time between 0 and t^* and with respect to each of the parameters ω and Ω between 0 and 1, and utilizing condition (4), we determine the rupture time

$$t^* = \begin{cases} B^{-1}(\sigma_0/\sigma)^n & \text{for } \sigma < \sigma_0, \\ B^{-1}(\sigma_0/\sigma)^m & \text{for } \sigma > \sigma_0. \end{cases} \tag{5}$$

which corresponds to a known approximation of experimental creep strength curves in double logarithm coordinates $\log\sigma - \log t^*$ in the form of two rectilinear sections (Fig. 1). The stress σ_0 corresponds to the intersection of these sections.

Let a constant stress σ_1 act for a time t_1, after which it changes by a jump to the value σ_2, and then remains invariant for a time t_2 up to rupture.

We first consider the case when each of the stresses σ_1 and σ_2 are greater than σ_0. An analysis of (3) shows that in this case the parameter Ω becomes 1 more rapidly than ω, independently of the value of t_1, as well as of whether σ_1 or σ_2 is greater. Hence, in this case just the second of the two equations in (3) is essential. It can be shown that condition (2) is always satisfied for $\min(\sigma_1, \sigma_2) > \sigma_0$. An analogous deduction also follows in the case that each of the stresses σ_1 and σ_2 is less than σ_0.

Now we consider a step loading when the passage through $\sigma = \sigma_0$ is performed for $t = t_1$. To do this we rewrite (3) in a form more convenient for investigation

$$df = B(\sigma/\sigma_0)^n\, dt,\ dF = B(\sigma/\sigma_0)^m dt. \tag{6}$$

We first investigate the case of partial unloading ($\sigma_1 > \sigma_0 > \sigma_2$). We introduce the dimensionless variables $x = \sigma_1/\sigma_0$, $y = \sigma_2/\sigma_0$ and we integrate (6). We consequently determine the time t_ω^* during which the parameter ω and its corresponding functions $f(\omega)$ reach the value t_Ω^* as well as the time Ω during which the parameter Ω and the function $F(\Omega)$ equal 1:

$$\begin{aligned} B\left[x^n t_1 + y^n (t_\omega^* - t_1)\right] = 1,\ t_\omega^* = t_1 + y^{-n}(B^{-1} - x^n t_1), \\ B\left[x^m t_1 + y^m (t_\Omega^* - t_1)\right] = 1,\ t_\Omega^* = t_1 + y^{-m}(B^{-1} - x^m t_1). \end{aligned} \tag{7}$$

The true rupture time t^* is defined as the minimum of the two values t_ω^* and t_Ω^*. There follows from a comparison of t_ω^* and t_Ω^*

$$t^* = \begin{cases} t_\omega^* & \text{for } t_1/t_1^* < c_1, \\ t_\Omega^* & \text{for } t_1/t_1^* > c_1, \end{cases} \tag{8}$$

and the value of c_1 is determined from the equation $c_1 = (1 - y^k)/(1 - y^k x^{-k})$. From (5) we obtain that the rupture time $t_1^* = (Bx^m)^{-1}$ corresponds to the stress x acting constantly, and the rupture time $t_2^* = (By^n)^{-1}$ to the stress y. Using (7) and (8), we calculate the sum of the partial times (1)

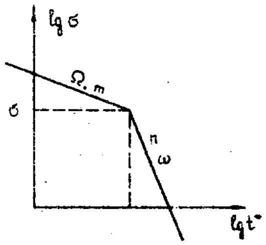

Fig. 1

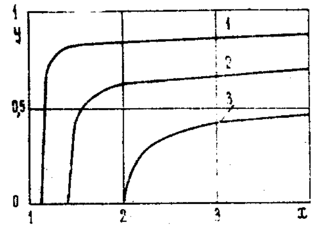

Fig. 2

$$A=\begin{cases}1+(t_1/t_1^*)(1-x^{-k}) & \text{for } t_1/t_1^*<c_1,\\ 1+(1-t_1/t_1^*)(y^{-k}-1) & \text{for } t_1/t_1^*>c_1.\end{cases} \quad (9)$$

It is seen from (9) that the quantity A always exceeds 1 independently of t_1/t_1^*. The dependence of A on t_1/t_1^* is a continuous piecewise-linear function that maps a two-piece line. The maximum of A is achieved for $t_1/t_1^* = c_1$, hence

$$A_{\max}=1+(1-x^{-k})(1-y^{k})/(1-y^{k}x^{-k}). \quad (10)$$

The value of A_{max} for a half-strip on the (x, y) plane can be found from (10): $x > 1$, $0 < y < 1$. On both boundaries ($x = 1$ and $y = 1$) of this half-strip $A_{max} = 1$. Along the ray ($x > 0$, $y = 0$) A_{max} increases as x grows asymptotically from 1 to 2. An analysis of (10) shows that the model (3) under consideration predicts a unilateral deviation from the linear summation principle when $\sigma_1 > \sigma_0 > \sigma_2$

$$1<A<2. \quad (11)$$

Investigation of the creep strength for $\sigma_1 < \sigma_0 < \sigma_2$ is performed in the same manner. In this case the rupture time t* is determined from the formula

$$t^*=\begin{cases}t_\Omega^*=t_1+y^{-m}(B^{-1}-x^{m}t_1) & \text{for } t_1/t_1^*<c_2,\\ t_\omega^*=t_1+y^{-n}(B^{-1}-x^{n}t_1) & \text{for } t_1/t_1^*>c_2,\end{cases}$$

where $c_2 = (1 - y^{-k})/(1 - x^{k}y^{-k})$. The sum of the partial times is calculated, as usual, from (1):

$$A=\begin{cases}1+(1-x^{k})(t_1/t_1^*) & \text{for } t_1/t_1^*<c_2,\\ 1+(y^{k}-1)(1-t_1/t_1^*) & \text{for } t_1/t_1^*>c_2.\end{cases}$$

It can be seen that for $x < 1 < y$ the sum of partial times always exceeds one, as in the previous case. In this case the inequality (11) is also satisfied. The maximal value of the sum A equals

$$A_{\max}=1+(1-x^{k})(1-y^{-k})/(1-x^{k}y^{-k}).$$

Let us examine the first quadrant in the (x, y) plane and let us isolate domains with different values of A. In the quadrant $(0\leqslant x\leqslant 1,\ 0\leqslant y\leqslant 1)$, as well as within the right angle $(x\geqslant 1,\ y\geqslant 1)$, A always equals 1 for any combinations of x and y. In the remaining half-strips ($x > 1$, $0 < y < 1$ and $0 < x < 1$, $y > 1$) A is between 1 and 2. Curves 1-3 corresponding to the constant levels A_{max} (1.25, 1.5, 1.75) in Fig. 2 are presented within the half-strip ($x > 1$, $0 < y < 1$) for the case $k = 2$. The analysis performed shows that the model (3) and (4) can describe a unilateral deviation from the principle of linear summation ($1 < A < 2$) only in cases when $\sigma_1 < \sigma_0 < \sigma_2$ or $\sigma_1 > \sigma_0 > \sigma_2$.

Let us turn to the case of multistep loading. If the loading is characterized by multiple alternation of the stresses σ_1 and σ_2, without passage through σ_0, then one of the two structure parameters being investigated is dominant; material rupture is determined by the behavior of just this structure parameter and the linear summation principle for the partial times is not satisfied identically.

Let us consider alternation of the stresses with passage throuth σ_0. We start from the unilateral loading presented above, which we divide into two stages with respect to time. Let the stress $\sigma_1 > \sigma_1$ act during $0.5t_1$, then the stress $\sigma_2 < \sigma_0$ is applied during $0.5t_2$. Afterwards additional loading ($\sigma_2 < \sigma_0$ during $0.5t_1$) occurs and then repeated unloading (the stress $\sigma_2 < \sigma_0$ is applied during τ until rupture). Integrating (6) for such loading, we obtain that the time τ of the stress σ_2 acting in the second cycle is $0.5t_2$. If a loading with many passages through σ_0 is considered, then it can be shown by an analogous method that rupture sets in because of the combined action of the stress σ_1 during the time t_1 and the stress σ_2 during t_2, where t_1 is the total time of application of the stress σ_1 in all the stages, and t_2 is the total time during which the stress σ_2 was applied. It hence follows that the sum of the partial times is independent of the quantity of passages of the stress through σ_0, and agrees with the value (9). Therefore, model (3) and (4) for the description of loading with single and multiple passages through the stress σ_0 results in a unilateral deviation ($1 < A < 2$) from the linear summation principle for the partial times.

Let us note that if the rupture condition (4) is replaced by

$$\min(\omega, \Omega) = 1, \tag{12}$$

then the deviation from condition (2) towards $A < 1$ can be described by using the model (3) and (12).

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PROBLEM OF NORMAL PRESSURE WAVES RUNNING AGAINST A STAMP

V. A. Babeshko, Zh. F. Zinchenko, and A. V. Smirnova

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The problem of the motion of a rigid massive circular stamp with a flat base under the action of oncoming normal pressure waves is examined. The stamp is assumed to be in frictionless contact with an elastic medium. It is assumed that the pressure wave is a plane wave and arrives from infinity. By removing the normal pressures from the surface of the medium by solving the boundary-value problem in the absence of the stamp (unmixed problem), the starting boundary-value problem reduces to the following mixed problem: a wave, which interacts with the stamp, travels along the surface of the medium screened from the normal pressure. Adding to the solution of this mixed problem the solution corresponding to the unmixed problem, we obtain the solution of the starting problem. Taking into account the fact that it is easy to solve the unmixed problem with the help of Fourier and Laplace integrals, in this work, we are primarily concerned with the mixed problem noted above with a screened surface outside the stamp.

1. We are studying the problem of the interaction of a rigid stamp with mass m, occupying a circular region Ω with radius a in a plane, with an elastic layered medium. It is assumed that the contact is frictionless, while a uniformly moving normal pressure pulse $p(x, y, t)$ acts on the stamp. It is necessary to find the normal component of the contact stresses $q(x, y, t)$, the vertical displacement of the center of the stamp $\delta(t)$, as well as the angles of its rotation relative to the horizontal axes $\omega(t)$ and $\theta(t)$; we shall determine $q(x, y, t)$ by solving the dynamic Lamb equation

$$(\lambda + 2\mu)\,\mathrm{grad}\,\mathrm{div}\,\mathbf{U} - \mu\,\mathrm{rot}\,\mathrm{rot}\,\mathbf{U} - \rho\partial^2\mathbf{U}/\partial t^2 = 0$$

with mixed boundary conditions and initial conditions. In particular, in the case of nonstationary action of the stamp on an elastic homogeneous half-space ($z \leqslant 0$), the boundary conditions have the form

$$\tau_{xz}(x, y, 0, t) = \tau_{yz}(x, y, 0, t) = 0, \quad -\infty < x, y < +\infty,$$

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